

Intermediate Algebra – Introduction to Visualization

Big Idea

Vocabulary

Prerequisite

- ☐ represent
- ☐ system of equations
- ☐ intersection
- ☐ equation
- ☐ expression
- ☐ commutative property of addition
- ☐ variable

New

- ☐ visualization
- ☐ arbitrary

Visualization

Dots

When you were learning the basic facts of math, you probably used objects to represent quantities. For example three marbles could represent the quantity 3. Add one more marble to the three, and you have represented the equation $3 + 1 = 4$. We can also do this using dots.

1. Try this. Draw three dots, a space and one dot. This is a representation of $3 + 1 = 4$.
2. Now draw a representation of $3 + 1 = 1 + 3$. Compare this with other students. Which representation is most clear.
3. Try drawing a representation of $2 \cdot 3 = 6$. Draw two rows of three dots.
4. Now use dots to represent $2 \cdot 3 = 3 \cdot 2$. Compare your results with other students.
5. How could you use dots to represent $4 - 1 = 3$?
6. Could you represent the number 3.14159 accurately with dots? Why or why not?

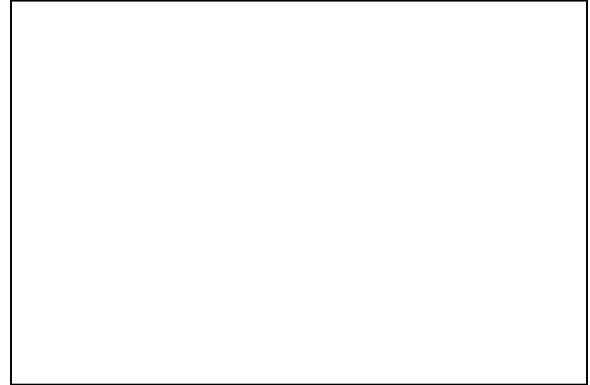
Measurement

Take a ruler and measure this line in inches: _____

7. Write your measurement here: _____

If the line's measurement is 2, we can say that the line *represents* the number 2.

8. Use your ruler to measure the dimensions of this rectangle. What is the area of the rectangle?
9. What math operation did you use to calculate the area of the rectangle (addition, subtraction, multiplication, or division)?
10. The rectangle can be said to represent what math fact?



11. Now write a representation of the algebraic expression $3 + 1$ using lines. How does this show that $3 + 1 = 4$?
12. Write a representation for the commutative property of addition using $3 + 1 = 1 + 3$.
13. Could you accurately represent the number 3.14159 using a line? Why or why not?
14. Could you accurately represent the number π using a line? Why or why not?
15. Compare these representations with other students. Which representation is the clearest?

Number Line

Number lines are often used to visualize addition and subtraction. Use the number line below to visualize $3 + 2 = 5$ and $3 - 7 = -4$.



Graphs

16. Graphs are used to visualize algebraic relationships. Below is a graph of $y = 2x - 3$. Use the graph to find the value of y when x is 2.

**INSERT GRAPH HERE **

17. Graphs can also be used to solve systems of equations. If we graph two equations, their intersection is the solution to the system of two equations. Below is a graph of $y = x^2$ and $y = x$. At what two points do the two lines intersect?

**INSERT GRAPH HERE **

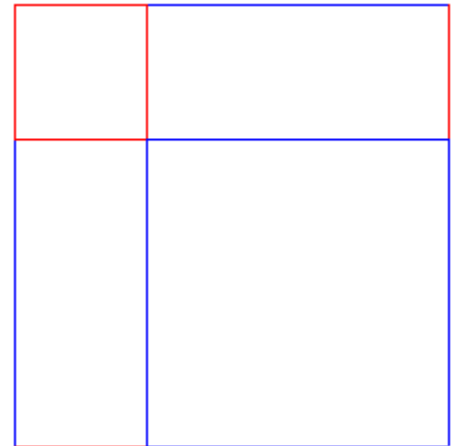
Visualization of $(a + b)^2 = a^2 + 2ab + b^2$

When we do not know the exact measure of a line, we can still represent it with an *arbitrary* line. This means the line can have any length. To represent the variable a , we might draw a line of length 1, for example. To represent the variable b , we might draw a line of length 2.

18. Draw a line representing $a + b$.

19. Since you can use a rectangle to represent multiplication, draw a rectangle to represent $(a + b) \cdot (a + b)$ or $(a + b)^2$. One side will be the line $a + b$. The other side will be the same.

20. Here is a figure representing $(a + b)^2$. Find one square of size a^2 , one of size b^2 , and two of size ab . Label each small rectangle with the area of that rectangle.



Other Resources

- David McAdams, Geometric Representation of $(a + b)^2 = a^2 + 2ab + b^2$, <http://mcadamsmath.tripod.com/algebra/apb2.html>, last accessed 12/29/2006.
- David McAdams, *Distributive Property*, <http://mcadamsmath.tripod.com/algebra/distributive.html>, last accessed 12/29/2006.
- David McAdams, *Geometric Representation in Algebra*, <http://mcadamsmath.tripod.com/algebra/introgeorep.html>, last accessed 12/29/2006